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Towards accurate modeling of the multidimensional MPI physics

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Abstract

The MPI image reconstruction problem requires, particularly for 2D and 3D excitation patterns, a measured system matrix due to the lack of an accurate model that is capable of describing the nanoparticles' magnetization behavior in the MPI setup. Here we exploit a model based on Néel rotation for large particle ensembles and we find model parameters that describe measured 2D MPI data with higher precision than state of the art MPI models, which is also illustrated in phantom experiments. This is a short summary of the recent work (4) to which we refer to for all further details.

I Introduction

Signal encoding in MPI is achieved by exposing the MNPs to one or several homogeneous excitation fields [6]. The MNPs respond with a change of their magnetization, which is measured with one or multiple receive coils. In ferrofluids the change of the particle magnetization is enabled by two different well studied dynamic processes, i.e. Brownian and Néel rotation of the magnetic moment. In the context of MPI these dynamics were initially studied by Weizenecker et al. [8]. A first attempt to Brownian rotation via the Fokker-Planck equation was presented by Yoshida and Enpuku for 1D excitation patterns [9] followed by further works in this direction (see the recent survey [3] and the references therein).

One prerequisite for MPI image reconstruction is that the system matrix is accurately known. Since its very first introduction in [1], the system matrix is measured in a data-driven calibration procedure. Drawbacks (time-consuming, noisy) motivated the development of methods determining the MPI system matrix based on physical models. While initial model-based results like in [5] were promising, model-based reconstruction based on the used *equilibrium model* [3] (denoted by *model A* in the remainder) leads to worse image quality when compared to the data driven approach. This is why the latter one is still the method of choice in almost all publications since 2010 that use multidimensional excitation patterns in the context of image reconstruction. For 1D excitation, which includes 2D and 3D Cartesian like sampling patterns, model-based reconstruction has been established in various works, e.g., as in [2].

Within this work we investigate the question why the equilibrium model fails to accurately describe the MPI system matrix for 2D Lissajous type excitation patterns. Fig. 1 illustrates the problem as one can identify various differences that lead to large numerical deviations.

In particular we want to highlight two qualitative effects: In outer regions of the field-of-view (FoV) $\Omega \subset \mathbb{R}^3$ wave peaks are merging and a tilting of the outer wave structures can be observed. The goal of this work is to simulate these effects using a physical model that takes Néel rotation into account.

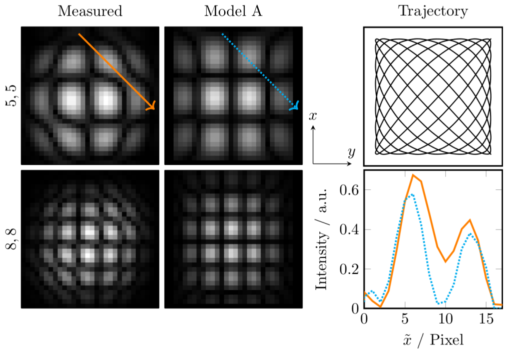

Figure 1: Absolute of two selected frequency components for mixing factors (1st/2nd row) for different system matrices (1st/2nd col.). The 3rd col. shows a 2D cosine sampling pattern (top) and line profiles (bottom) along the marked path.

II Methods

The investigated *model B* is derived from the behavior of an entire ensemble of nanoparticles where the mean magnetic moment is determined via the probability density function $f : S^2 \times \Omega \times [0, T] \to \mathbb{R}_0^+$, which is the solution to the corresponding Fokker-Planck equation where S^2 is the surface of the sphere in $\mathbb{R}^3$ and $T > 0$ is the measurement time. The mean is then given by $\bar{m}(x,t) = m_0 \int_{S^2} m f(m,x,t) dm$ where f is the solution to the following PDE on the sphere

$$\frac{\partial}{\partial t} f = \text{div}_{S^2}\left(\frac{1}{2\tau} \nabla_{S^2} f\right) - \text{div}_{S^2}(bf) \quad (1)$$

where $\tau > 0$ is the relaxation time constant and the (velocity) field $b : S^2 \times \mathbb{R}^3 \times S^2 \to \mathbb{R}^3$ given by

$$b(m, H_{app}, n) = p_1 H_{app} \times m + p_2 (m \times H_{app}) \times m + p_3 (n \cdot m) n \times m + p_4 (n \cdot m)(m \times n) \times m \quad (2)$$

where $pi \geq 0$, $i = 1, \ldots, 4$, are physical constants and $n \in S^2$ is the easy axis of the particles. Here we distinguish the following three cases, i.e., Néel rotation model

- B1 without anisotropy, i.e., $p_3 = p_4 = 0$.
- B2 including anisotropy, i.e., it includes all summands in (2) for a given easy axis $n \in S^2$.
- B3 including the model assumption of a space-dependent anisotropy, which is related to the local structure of the magnetic field, i.e., $n : \Omega \to S^2$ and $p_3, p_4 : \Omega \to \mathbb{R}_0^+$. Here, we use $n(x) = \frac{H_S(x)}{|H_S(x)|}$ and $K_{anis}(x) = \frac{g_{K_{anis}} |H_S(x)|}{h}$ with anisotropy gradient $g_{K_{anis}}$ and $h = max_{x \in \Omega} |H_S(x)|$ resulting in p_3, p_4.

III Results

Results from the spatially homogeneous anisotropy distribution (model B2) motivate the usage of a space-dependent anisotropy distribution (model B3). Note that distribution is meant with respect to the FoV Ω. In order to make the tilting orientation-dependent, we let the easy axis be parallel to the selection field vector. The results are shown in Fig. 2. One can see that the introduction of the spatially inhomogeneous easy axis and anisotropy constant does indeed introduce the two effects observed in the measurements.

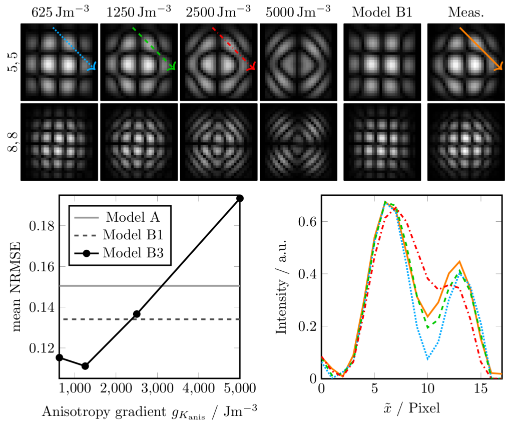

Figure 2: Model B3 with anisotropy gradients $g_{K_{anis}}$ (cols. 1-4). Col. 5 shows the modeled system matrix using the model B1 while col. 6 shows the measured system matrix for reference. Below the mean NRMSE between the model and the measurement (left) and line profiles along the marked path are shown.

Reconstruction results for phantom data can be found in [4, Fig. 6]. There one can see that the Néel model achieves a similar image quality as the calibrated system matrix when applying a certain type of frequency selection. Model A shows a much worse image quality, which underlines, why it is usually not used in practice.

IV Conclusions

The results show that the physics of a multidimensional MPI experiment can be approximately modeled by solving the Fokker-Planck equation considering Néel rotation with a spatially inhomogeneous anisotropy. One can expect that the accuracy of the model increases even further when considering a particle size distribution obtained by an optimization procedure. However, the proposed model can be rather interpreted as an approximate model than a physical model. A full physical model needs to take into account an anisotropy constant distribution as well as a time-dependent distribution of the particles' easy axis. The latter distribution would for example be a result of a full physical model taking into account the coupling of Brownian and Néel rotation in ferrofluids. The relationship between a full physical model and the proposed approximate model remains future research. The results of model B2 show the potential for an encoding of orientation information when using for example im-

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mobilized nanoparticle markers characterized by a dominant orientation. In the context of multi-contrast MPI [7] this may build a new dimension of contrast, which remains future research.

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