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Towards Addressing Nanoparticle Relaxation in Model-Based Reconstruction

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Abstract

Magnetic Particle Imaging (MPI) is a tracer-based imaging modality that uses superparamagnetic nanoparticles as tracer. The Langevin model is a simple model that is capable of capturing the overall relation between the measured signal and the particles, but it ignores relaxation effects. Relaxation effects can be accounted for in the Debye model, which introduces a relaxation parameter τ, representing the delay in the alignment of the nanoparticles to the magnetic field. Starting from the Debye model, we propose a relaxation adaption step to account for the delay τ in model-based reconstructions that use the Langevin model.

I. Introduction

During an MPI scan, the magnetic nanoparticles constantly realign their magnetic moments to match the dynamic magnetic field applied. In the Langevin model [1], the realignment is assumed to be instantaneous (adiabatic assumption). However, real nanoparticles do not behave ideally and are influenced by thermal fluctuations as well as by viscous resistance. These influences cause delays in the magnetization response, a phenomenon called relaxation [2]. To account for the particles' behavior, models incorporating Néel and Brownian rotation have been considered and simulations based on the Fokker-Plank equation have been conducted [3]. The usage of more complex models is non-trivial and comes at a higher computational effort [4]. The Debye model [2, 5] offers a rather simple way of modeling relaxation and can be more easily combined with existing approaches. We introduce a relaxation adaption step to account for the relaxation in the two-stage algorithm proposed and further developed by the authors in [6, 7].

II. Methods and materials

If ρ is a particle concentration, the background-corrected signal $s(t)$ during a scan can be modeled as [3]

$$s(t) = -\mu_0 \frac{d}{dt}\left[\int_{\mathbb{R}^n} R(x)M(x,t)\,dx\right], \tag{1}$$

where μ_0 is the magnetic permeability in vacuum, $M(x,t)$ is the magnetization response of the particles and $R(x)$ is the response coils' sensitivity pattern (assumed $R(x) \equiv \mathbb{I}_n$ for simplicity). In the Langevin model (adiabatic), the magnetization response is $M_{ad}(x,t) = m\rho(x)\mathcal{L}\left(\frac{\|H(x,t)\|_2}{H_{\text{sat}}}\right)\frac{H(x,t)}{\|H(x,t)\|_2}$, where $\mathcal{L} = \cosh(z) - \frac{1}{z}$ is the Langevin function, $H(x,t)$ is the applied magnetic field and H_{sat} is a saturation parameter. In the field-free point (FFP) MPI, the filed is $H(x,t) = G(x - r(t))$ where G is the gradient of the static field and $r(t)$ is the trajectory of the FFP [8]. In the Langevin model, the signal $s(t)$ is [8]

$$s(t) = A_{H_{\text{sat}}}[\rho](r(t))\, v(t), \tag{2}$$

where $v(t) = \frac{d}{dt}r(t)$ and $A_{H_{\text{sat}}} \colon L^1(\mathbb{R}^n) \mapsto C_b^\infty(\mathbb{R}^{n\times n})$ is the MPI Core Operator defined by the convolution $A_{H_{\text{sat}}}[\rho](x) = (K_{H_{\text{sat}}} * \rho)(x)$ with the MPI Kernel $K_{H_{\text{sat}}}(y)$. We have shown in [6], that the concentration ρ can be reconstructed using a two-stage approach. Starting from the input data $s_k = s(t_k)$, $r_k = r(t_k)$ and $v_k = v(t_k)$ collected a times $t_k = k/f$ for $k = 0, 1, \dots, (L-1)$ with sampling frequency f, one can use (2) to obtain $A \in \mathbb{R}^{N_x \times N_y \times 2\times 2}$ approximating $A_{H_{\text{sat}}}[\rho]$ on a $N_x \times N_y$ grid by solving

$$A = \arg\min_{\hat{A}} \left\{ \frac{1}{L} \sum_{k=0}^{L-1} \left\| s_k - \mathrm{I}[\hat{A}](r_k) v_k \right\|_2^2 + \gamma \mathscr{R}_C[\hat{A}] \right\}, \tag{3}$$

for a regularizer $\mathscr{R}_C$ and parameter $\gamma > 0$. Solution of (3) is the *MPI Core Stage* [7]. Once A is obtained, in the *Deconvolution Stage* we deconvolve $A \approx A_{H_{\text{sat}}}[\rho]$ with $K_{H_{\text{sat}}}$.

We consider a first-order Debye process [2] given by

$$\frac{dM(x,t)}{dt} = -\frac{M(x,t) - M_{ad}(x,t)}{\tau} \tag{4}$$

for a relaxation time constant $\tau > 0$. The inclusion of τ in our reconstruction framework yields the possibility of accounting for relaxation effect by means of a relaxation adaption step. We present here the relaxation adaption step; the details and derivations are topic of an upcoming publication.

With $\alpha = \exp(-\frac{1}{f\tau})$, the *relaxation adaption* steps is

$$\tilde{s}_n = \frac{s_n - \alpha s_{n-1}}{1-\alpha}. \tag{5}$$

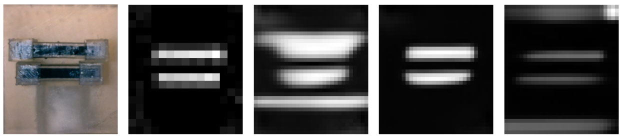
With the correction in (5), the model of the adapted signal is $\tilde{s}_n = A_{H_{\text{sat}}}[\rho](r_n)\, v_n$, as in (2). Consequently, the reconstruction scheme proposed is:

- *Relaxation adaption*: convert the input data s_n into $\tilde{s}_n$ using (5).
- *MPI Core Stage*: taking $\tilde{s}_n$ as inputs, reconstruct A using (3).
- *Deconvolution Stage*: deconvolve u using $K_{H_{\text{sat}}}$.

We test the relaxation adaption on real 2D MPI data.

III. Results and discussion

We consider the resolution 1 phantom (fig. 1a) in the "MPIData: Equilibrium Model with Anisotropy" (EMWA) dataset [9], scanned with the preclinical Bruker scanner (Ettlingen, Germany). We reconstruct the phantom with our model-based algorithm in [7] first without relaxation adaptation ($\tau = 0$ s) and then with the adaptation using $\tau = 1 \times 10^{-6}$ s and $\tau = 1 \times 10^{-5}$ s. The choice of $\tau = 1 \times 10^{-6}$ s (or 1 µs) is coherent with the measured relxation parameter of nanoparticles, which is in the order of microseconds [10]. The reconstructions have been performed in the 34 mm × 30 mm FoV in which also a



(a) Phantom (b) SM reco. (c) $\tau = 0$ (d) $\tau = 10^{-6}$ (e) $\tau = 10^{-5}$

Figure 1: Reconstruction results on the resolution phantom (1a). (1b) Reconstruction with the system matrix provided with [9]; (1c) model-based reconstruction without relaxation adaption [7]; (1d-1e) model-based reconstruction with the relaxation adaption in (5) with 1 µs (1d) and 10 µs (1e).

system-matrix has been calibrated. We point out that the DF-FoV where the data is collected is smaller and cover the central 24 mm × 24 mm region. We use the provided system matrix to reconstruct the phantom (fig. 1b) using the Kaczmarz algorithm [11] for comparison with our method, as the system matrix natively accounts for the complex behavior of the particles. From a comparison of the results in figure 1, we can see that the inclusion of the relaxation adaption step proposed, helps obtaining a model-based reconstruction (fig. 1d) which is closer to the relaxation-accounting system-matrix-based reconstruction (fig.1b), and presents less artifacts of the reconstruction obtained with the Langevin model without relaxation adaption (fig. 1c). However, if the correction is too strong (fig. 1e), the reconstruction presents again artifacts in the final reconstruction.

IV. Conclusion

In this contribution we have proposed a relaxation adaption to account for the relaxation of real magnetic nanoparticles as per the Debye model. The relaxation adaption step can be used in conjunction with Langevin-based schemes that perform the MPI Core Stage and the Deconvolution Stage. In this way, the computational complexity of the solver used is not increased, and the relaxation is accounted for by applying (5).

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Author's statement

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